The Derivative of a function(函数的导数) (p99)

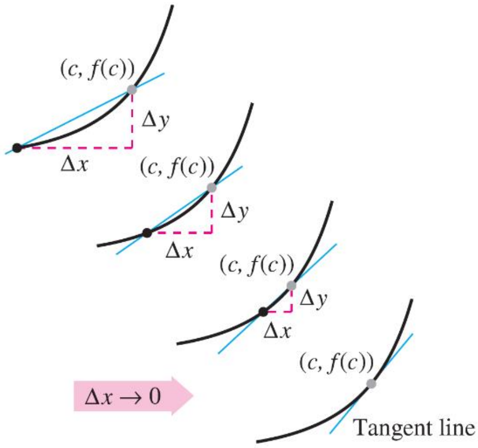
1. The derivative of function *f(x)* at *x* is given by(x 点的导数)

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

2. For all *x* for which this limit exists, $f'(x)$ is a function of *x*.(导函数)
3. Notation for derivatives

$$f'(x), \quad \frac{dy}{dx}, \quad y',$$

$$\frac{d}{dx}[f(x)], \quad D_x[y]$$



The function $f'(x)$ is read as "*f-prime of x*".

Differential of a function(函数的微分)

The differential is defined by

$$dy = f'(x)dx$$

The notion dy/dx is read as the *derivative of y with respect to x* or *dy,dx*

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} = f'(x)$$

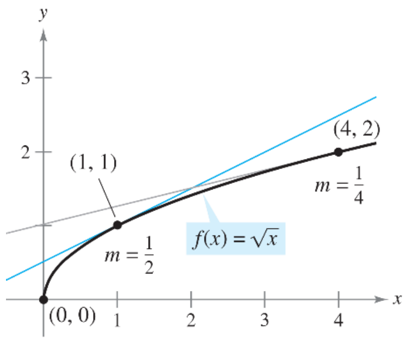
Example 1: Using the derivative to find the slop at a point (p100)

Find $f'(x)$ for $f(x) = \sqrt{x}$. Then find the slopes of the graph of $f(x)$ at the point $(1,1)$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} = \frac{1}{2\sqrt{x}}$$

$$(x > 0)$$

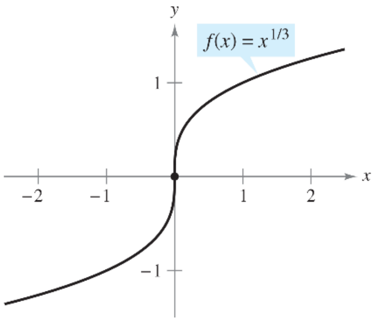
$$f'(1) = \frac{1}{2}$$

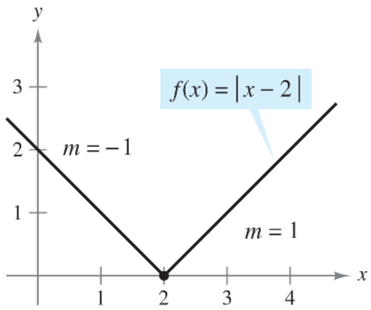


Differentiability and continuity(可导与连续) (p101)

Differentiability ⇒ Continuity

可导必然连续，连续不一定可导 Although it is true that differentiability implies continuity, the converse is no true.





$$\lim_{x \to 0^-} (x^{1/3}) = \lim_{x \to 0^+} (x^{1/3})$$

$$\lim_{x \to 2^-} |x-2| \neq \lim_{x \to 2^+} |x-2|$$

基本微分公式

不用记也不会忘的： 3

$\frac{d}{dx}[c]=0$　　$\frac{d}{dx}[kx]=k$　　$\frac{d}{dx}[kf(x)]=kf'(x)$

基本函数 3

$\frac{d}{dx}[x^n]=nx^{n-1}$　　$\frac{d}{dx}[\sin x]=\cos x$　　$\frac{d}{dx}[\cos x]=-\sin x$

其他三角函数 2+2

$\frac{d}{dx}[\tan x]=\sec^2 x$　　$\frac{d}{dx}[\cot x]=-csc^2 x$

$\frac{d}{dx}[\sec x]=\sec x\tan x$　　$\frac{d}{dx}[\csc x]=-\csc x\cot x$

基本运算 3

$\frac{d}{dx}[u\pm v]=u'\pm v'$　　$\frac{d}{dx}[uv]=vu'+uv'$　　$\frac{d}{dx}\left[\frac{u}{v}\right]=\frac{vu'-uv'}{v^2}$

复合函数 2

$\frac{dy}{dx}=\frac{dy}{du}\cdot\frac{du}{dx}$　　$\frac{d}{dx}[u^n]=nu^{n-1}u'$

Exercise 1: Find the derivative by the limit process.

1.1 $f(x)=7$

1.2 $f(x)=x^3-3x$

1.3 $f(x)=\frac{7}{x-1}$

1.4 $f(x)=\frac{1}{x^2}$

1.5 $f(x)=\sqrt{x+4}$

1.6 $f(x)=\frac{4}{\sqrt{x}}$

Exercise 2: (p104-35)Find an equation of the line that is tangent to the graph of $f(x)$ and parallel to the given line.

Function	Line
35. $f(x)=x^3$	$3x-y+1=0$
37. $f(x)=\frac{1}{\sqrt{x}}$	$x+2y-6=0$
38. $f(x)=\frac{1}{\sqrt{x-1}}$	$x+2y+7=0$

Exercise3:(P106-105)Let $f(x)=\begin{cases} x\sin\frac{1}{x}, & x\neq 0 \\ 0, & x=0 \end{cases}$ and $g(x)=\begin{cases} x^2\sin\frac{1}{x}, & x\neq 0 \\ 0, & x=0 \end{cases}$.

Show that *f* is continuous, but not differentiable, at $x=0$.

Show that *g* is differentiable at *0*, and find $g'(0)$.

Exercise 4 : (p126-1) Use the Product Rule to differentiate the function.

1. $g(x)=(x^2+3)(x^2-4x)$

2. $f(x)=(6x+5)(x^3-2)$

3. $h(t)=\sqrt{t}(1-t^2)$

4. $g(s)=\sqrt{s}(s^2+8)$

5. $f(x)=x^3\cos x$

6. $g(x)=\sqrt{x}\sin x$

Exercise 5 : (p126-7)Use the Quotient Rule to differentiate the function.

7. $f(x)=\frac{x}{x^2+1}$

8. $g(t)=\frac{t^2+4}{5t-3}$

9. $h(x)=\frac{\sqrt{x}}{x^3+1}$

10. $h(s)=\frac{s}{\sqrt{s}-1}$

11. $g(x)=\frac{\sin x}{x^2}$

12. $f(t) = \frac{\cos t}{t^3}$

Exercise 6: (p126-34)Find the derivative of the algebraic function.

34. $g(x) = x^2\left(\frac{2}{x} - \frac{1}{x+1}\right)$

35. $f(x) = (2x^3 + 5x)(x - 3)(x + 2)$

37. $f(x) = \frac{x^2+c^2}{x^2-c^2}$, c is a constant

38. $f(x) = \frac{c^2-x^2}{c^2+x^2}$, c is a constant

Exercise 7: (p126-40)Find the derivative of the trigonometric function.

40. $f(\theta) = (\theta + 1)\cos\theta$

42. $f(x) = \frac{\sin x}{x^3}$

44. $y = x + \cot x$

46. $h(x) = \frac{1}{x} - 12\sec x$

48. $y = \frac{\sec x}{x}$

50. $y = x\sin x + \cos x$

52. $f(x) = \sin x \cos x$

54. $h(\theta) = 5\theta\sec\theta + \theta\tan\theta$

Exercise 8: (p127-73)Determine the point(s) at which the graph of the function has a horizontal tangent line.

73. $f(x) = \frac{2x-1}{x^2}$

74. $f(x) = \frac{x^2}{x^2+1}$

75. $f(x)=\frac{x^2}{x-1}$

76. $f(x)=\frac{x-4}{x^2-7}$

Exercise 2:

35. $y = 3x - 2, y = 3x + 2$

37. $y = \frac{-1}{2}x + \frac{3}{2}$

38. $y = \frac{-1}{2}x + 2$

Exercise3:(P106-105)Let $f(x) = \begin{cases} x\sin\frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$ and $g(x) = \begin{cases} x^2\sin\frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$.

(1) Show that f is continuous, but not differentiable, at $x = 0$.

Squeeze theorem: $-|x| \leq x\sin\frac{1}{x} \leq |x|, \lim_{x\to 0}(-|x|) = 0, \lim_{x\to 0}(|x|) = 0,$

so that $\lim_{x\to 0}\left(x\sin\frac{1}{x}\right) = 0 = f(0)$, so f is continuous at $x = 0$

$$f'(x) = \lim_{x\to 0}\frac{f(x) - f(0)}{x - 0} = \lim_{x\to 0}\frac{x\sin(1/x) - 0}{x - 0} = \lim_{x\to 0}\sin\frac{1}{x} \Rightarrow \text{limit doesn't exist.}$$

(2) Show that g is differentiable at *0*, and find $g'(0)$.

Squeeze theorem: $-x^2 \leq x^2\sin\frac{1}{x} \leq x^2, \lim_{x\to 0}(-x^2) = 0, \lim_{x\to 0}(x^2) = 0,$

so that $\lim_{x\to 0}\left(x^2\sin\frac{1}{x}\right) = 0 = f(0)$, so f is continuous at $x = 0$

$$g'(x) = \lim_{x\to 0}\frac{g(x) - g(0)}{x - 0} = \lim_{x\to 0}\frac{x^2\sin(1/x) - 0}{x - 0} = \lim_{x\to 0}x\sin\frac{1}{x} = 0$$

Therefore, $x = 0$ is differentiable at $x = 0, g'(0) = 0$.

Exercise 4:

1. $g'(x) = 4x^3 - 12x^2 + 6x - 12$
2. $f'(x) = 24x^3 + 15x^2 - 12$
3. $h'(t) = \frac{-5}{2}t^{3/2} + \frac{1}{2}t^{-1/2}$
4. $g'(s) = \frac{5}{2}s^{3/2} + 4s^{-1/2}$
5. $f'(x) = 3x^2\cos x - x^3\sin x$
6. $g'(x) = \frac{1}{2}x^{-1/2}\sin x + \sqrt{x}\cos x$

Exercise 5:

7. $f'(x) = \frac{-x^2+1}{(x^2+1)^2}$
8. $g'(x) = \frac{5t^2-6t-20}{(5t-3)^2}$
9. $h'(x) = \frac{\frac{-5}{2}x^{\frac{5}{2}}+\frac{1}{2}x^{\frac{-1}{2}}}{(x^3+1)^2}$
10. $h'(s) = \frac{\frac{1}{2}\sqrt{s}-1}{(\sqrt{s}-1)^2}$
11. $g'(x) = \frac{x\cos x-2\sin x}{x^3}$
12. $f'(t) = \frac{-t\sin t-2\cos t}{t^4}$

Exercise 6:

34. $g'(x) = 1 + \frac{1}{(x+1)^2}$

35. $f'(x) = 10x^4 - 8x^3 - 21x^2 - 10x - 30$

37. $f'(x) = \frac{-4c^2x}{(x^2 - c^2)^2}$

38. $f'(x) = \frac{-4c^2x}{(c^2 + x^2)^2}$

Exercise 7:

40. $f'(\theta) = \cos\theta - (\theta + 1)\sin\theta$

42. $f'(x) = \frac{\cos x}{x^3} - \frac{3\sin x}{x^2}$

44. $y' = 1 - \csc^2 x$

46. $h'(x) = \frac{-1}{x^2} - 12\sec x \tan x$

48. $y' = \frac{\sec x \tan x}{x} - \frac{\sec x}{x^2}$

50. $y' = x\cos x$

52. $f'(x) = \cos^2 x - \sin^2 x$

54. $h'(\theta) = 5\sec\theta + 5\theta\sec\theta\tan\theta + \tan\theta + \theta\sec^2\theta$

Exercise 8:

73. $f(x) = \frac{2x-1}{x^2}$ 非奇非偶函数, Domain: $x \neq 0$

a) $f(x) = \frac{2x-1}{x^2} = 2x^{-1} - x^{-2}$

b) $f'(x) = -\frac{2}{x^2} + \frac{2}{x}^{3} = 0 \Rightarrow x = 1$

c) $f(1) = \frac{2-1}{1} = 1 \Rightarrow at\ (1,1)\ f(x)$ has a horizontal tangent line.

74. $f(x) = \frac{x^2}{x^2+1}$ 偶函数, Domain: $x \in R$

a) $f(x) = \frac{x^2}{x^2+1} = \frac{x^2+1-1}{x^2+1} = 1 - \frac{1}{x^2+1} = 1 - (x^2+1)^{-1}$

b) $f'(x) = -(-1)(x^2+1)^{-2}(2x) = \frac{2x}{(x^2+1)^2} = 0 \Rightarrow x = 0$

c) $f(1) = 1/2$ at $(1, 1/2)$ has a horizontal tangent line.

75. $f(x) = \frac{x^2}{x-1}$ 非奇非偶函数, Domain: $x \neq 1$

a) $f(x) = \frac{x^2}{x-1} = \frac{x^2-1+1}{x-1} = \frac{(x-1)(x+1)}{x-1} + \frac{1}{x-1} = (x+1) + (x-1)^{-1}$

b) $f'(x) = 1 + (-1)(x-1)^{-2} = 1 - \frac{1}{(x-1)^2} = 0 \Rightarrow x = 2\ or\ x = 0$

c) $f(2) = 4, f(0) = 0 \Rightarrow at\ (2,4)\ , (0,0)\ f(x)$ has horizontal tangent lines.

76. $f(x) = \frac{x-4}{x^2-7}$ 非奇非偶函数, Domain: $x \neq \pm\sqrt{7}$

a) $f(x) = \frac{x-4}{x^2-7}$

b) $f'(x) = \frac{x^2-7-(x-4)2x}{(x^2-7)^2} = \frac{x^2-7-2x^2+8x}{(x^2-7)^2} = \frac{-x^2+8x-7}{(x^2-7)^2} = \frac{-(x-1)(x-7)}{(x^2-7)^2} = 0$

c) $\Rightarrow x = 1 \; or \; x = 7$

d) $\Rightarrow f(1) = \frac{1}{2} \; f(7) = \frac{1}{14}$

$\Rightarrow at \left(1, \frac{1}{2}\right), \left(7, \frac{1}{14}\right) \; f(x)$ has horizontal tangent lines.